

TWO SHARP INEQUALITIES FOR BOUNDING THE SEIFFERT MEAN BY THE ARITHMETIC, CENTROIDAL, AND CONTRA-HARMONIC MEANS

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ABSTRACT. In the paper, the authors find the best possible constants appeared in two inequalities for bounding the Seiffert mean by the linear combinations of the arithmetic, centroidal, and contra-harmonic means.

1. INTRODUCTION

For $a, b > 0$ with $a \neq b$, the Seiffert mean $T(a, b)$ and the centroidal mean $\overline{C}(a, b)$ are defined respectively by

$$T(a, b) = \frac{a - b}{2 \arctan\left(\frac{a-b}{a+b}\right)} \quad (1.1)$$

and

$$\overline{C}(a, b) = \frac{2(a^2 + ab + b^2)}{3(a + b)}. \quad (1.2)$$

It is well known that

$$A(a, b) = \frac{a + b}{2}, \quad G(a, b) = \sqrt{ab}, \quad S(a, b) = \sqrt{\frac{a^2 + b^2}{2}},$$

$$C(a, b) = \frac{a^2 + b^2}{a + b}, \quad M_p(a, b) = \sqrt[p]{\frac{a^p + b^p}{2}}$$

for $p \neq 0$ are respectively the arithmetic, geometric, root-square, contra-harmonic and p -th power means of two positive numbers a and b , that the p -th power means $M_p(a, b)$ is continuous and strictly increasing with respect to $p \in \mathbb{R}$ for fixed $a, b > 0$ with $a \neq b$, and that the inequalities in

$$G(a, b) = M_0(a, b) < A(a, b) = M_1(a, b) < \overline{C}(a, b) < S(a, b) = M_2(a, b) < C(a, b) \quad (1.3)$$

hold for $a, b > 0$ with $a \neq b$. For more information on results of mean values, please refer to, for example, [11, 12, 19, 20, 21] and closely related references therein.

In [22], Seiffert proved the double inequality

$$A(a, b) = M_1(a, b) < T(a, b) < M_2(a, b) = S(a, b) \quad (1.4)$$

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for $a, b > 0$ with $a \neq b$. In [13], Hästö showed that the function $\frac{T(1,x)}{M_p(1,x)}$ is increasing with respect to $x \in (0, \infty)$ if $p \leq 1$. In [3, 5], the authors demonstrated that the double inequalities

$$\alpha_1 S(a, b) + (1 - \alpha_1) A(a, b) < T(a, b) < \beta_1 S(a, b) + (1 - \beta_1) A(a, b) \quad (1.5)$$

and

$$\begin{aligned} C(\alpha_2 a + (1 - \alpha_2)b, \alpha_2 b + (1 - \alpha_2)a) &< T(a, b) \\ &< C(\beta_2 a + (1 - \beta_2)b, \beta_2 b + (1 - \beta_2)a) \end{aligned} \quad (1.6)$$

hold for $a, b > 0$ with $a \neq b$ if and only if

$$\alpha_1 \leq \frac{4 - \pi}{(\sqrt{2} - 1)\pi}, \quad \beta_1 \geq \frac{2}{3}, \quad \alpha_2 \leq \frac{1}{2} \left(1 + \sqrt{\frac{4}{\pi} - 1} \right), \quad \beta_2 \geq \frac{3 + \sqrt{3}}{6}. \quad (1.7)$$

For more information on this topic, please refer to recently published papers [4, 6, 7, 8, 9, 10, 14, 15, 16, 18, 23, 24] and cited references therein.

For positive numbers $a, b > 0$ with $a \neq b$, let

$$J(x) = \overline{C}(xa + (1 - x)b, xb + (1 - x)a) \quad (1.8)$$

on $[\frac{1}{2}, 1]$. It is not difficult to directly verify that $J(x)$ is continuous and strictly increasing on $[\frac{1}{2}, 1]$ and to notice that

$$J\left(\frac{1}{2}\right) = A(a, b) < T(a, b) \quad \text{and} \quad J(1) = \overline{C}(a, b) > T(a, b). \quad (1.9)$$

Therefore, it is much natural to ask a question: What are the best constants $\alpha \geq \frac{1}{2}$ and $\beta \leq 1$ such that the double inequality

$$\overline{C}(\alpha a + (1 - \alpha)b, \alpha b + (1 - \alpha)a) < T(a, b) < \overline{C}(\beta a + (1 - \beta)b, \beta b + (1 - \beta)a) \quad (1.10)$$

holds for $a, b > 0$ with $a \neq b$?

The following Theorem 1.1, the first main result of this paper, gives an affirmative answer to this question.

Theorem 1.1. *For positive numbers $a, b > 0$ with $a \neq b$, the double inequality (1.10) is valid if and only if*

$$\alpha \leq \frac{1}{2} \left(1 + \sqrt{\frac{12}{\pi} - 3} \right) \quad \text{and} \quad \beta = 1. \quad (1.11)$$

In [17] the author posed an unsolved problem: Find the greatest value α_1 and the least value β_1 such that the double inequality

$$\alpha_1 C(a, b) + (1 - \alpha_1) A(a, b) < T(a, b) < \beta_1 C(a, b) + (1 - \beta_1) A(a, b) \quad (1.12)$$

holds for $a, b > 0$ with $a \neq b$.

The following Theorem 1.2, the second main result of this paper, solves this problem.

Theorem 1.2. *for $a, b > 0$ with $a \neq b$, the double inequality (1.12) holds if and only if $\alpha_1 \leq \frac{4}{\pi} - 1$ and $\beta_1 \geq \frac{1}{3}$.*

2. PROOF OF THEOREM 1.1

In this section, we supply a proof of Theorem 1.1.

For simplicity, we denote two numbers in (1.11) by λ and μ respectively.

It is clear that, in order to prove the double inequality (1.10), it suffices to show

$$T(a, b) > \overline{C}(\lambda a + (1 - \lambda)b, \lambda b + (1 - \lambda)a) \quad (2.1)$$

and

$$T(a, b) < \overline{C}(\mu a + (1 - \mu)b, \mu b + (1 - \mu)a). \quad (2.2)$$

From definitions (1.1) and (1.2) we see that both $T(a, b)$ and $\overline{C}(a, b)$ are symmetric and homogenous of degree 1. Hence, without loss of generality, we assume that $a > b$. If replacing $\frac{a}{b} > 1$ by $t > 1$ and letting $p \in (\frac{1}{2}, 1)$, then

$$\begin{aligned} & \overline{C}(pa + (1 - p)b, pb + (1 - p)a) - T(a, b) \\ &= \frac{[pt + (1 - p)]^2 + [pt + (1 - p)][p + (1 - p)t] + [p + (1 - p)t]^2}{6(1 + t) \arctan \frac{t-1}{t+1}} b f(t), \end{aligned} \quad (2.3)$$

where

$$\begin{aligned} f(t) &= 4 \arctan \frac{t-1}{t+1} \\ &\quad - \frac{3(t^2 - 1)}{[pt + (1 - p)]^2 + [pt + (1 - p)][p + (1 - p)t] + [p + (1 - p)t]^2}. \end{aligned} \quad (2.4)$$

Standard computations lead to

$$f(1) = 0, \quad (2.5)$$

$$\lim_{t \rightarrow \infty} f(t) = \pi - \frac{3}{p^2 - p + 1}, \quad (2.6)$$

and

$$f'(t) = \frac{f_1(t)}{h_1(t)}, \quad (2.7)$$

where

$$\begin{aligned} f_1(t) &= (4p^4 - 8p^3 + 18p^2 - 14p + 1)t^4 - 4(4p^4 - 8p^3 + 9p^2 - 5p + 1)t^3 \\ &\quad + 6(4p^4 - 8p^3 + 6p^2 - 2p + 1)t^2 - 4(4p^4 - 8p^3 + 9p^2 - 5p + 1)t \\ &\quad + 4p^4 - 8p^3 + 18p^2 - 14p + 1, \end{aligned} \quad (2.8)$$

$$f_1(1) = 0, \quad (2.9)$$

and

$$h_1(t) = \{[pt + (1 - p)]^2 + [pt + (1 - p)][p + (1 - p)t] + [p + (1 - p)t]^2\}^2 (1 + t^2).$$

Let

$$f_2(t) = \frac{f_1'(t)}{4}, \quad f_3(t) = \frac{f_2'(t)}{3}, \quad \text{and} \quad f_4(t) = \frac{f_3'(t)}{2}.$$

Then, by standard argument, we have

$$\begin{aligned} f_2(t) &= (4p^4 - 8p^3 + 18p^2 - 14p + 1)t^3 - 3(4p^4 - 8p^3 + 9p^2 - 5p + 1)t^2 \\ &\quad + 3(4p^4 - 8p^3 + 6p^2 - 2p + 1)t - (4p^4 - 8p^3 + 9p^2 - 5p + 1), \end{aligned} \quad (2.10)$$

$$f_2(1) = 0, \quad (2.11)$$

$$f_3(t) = (4p^4 - 8p^3 + 18p^2 - 14p + 1)t^2 - 2(4p^4 - 8p^3 + 9p^2 - 5p + 1)t + 4p^4 - 8p^3 + 6p^2 - 2p + 1, \quad (2.12)$$

$$f_3(1) = 6p^2 - 6p, \quad (2.13)$$

$$f_4(t) = (4p^4 - 8p^3 + 18p^2 - 14p + 1)t - (4p^4 - 8p^3 + 9p^2 - 5p + 1), \quad (2.14)$$

$$f_4(1) = 9p^2 - 9p. \quad (2.15)$$

If $p = \lambda$, then the quantities (2.6), (2.13), and (2.15) become

$$\lim_{t \rightarrow \infty} f(t) = 0, \quad (2.16)$$

$$f_3(1) = \frac{18}{\pi} - 6 < 0, \quad (2.17)$$

$$f_4(1) = \frac{27}{\pi} - 9 < 0, \quad (2.18)$$

and

$$4p^4 - 8p^3 + 18p^2 - 14p + 1 = \frac{36 + 18\pi - 9\pi^2}{\pi^2} > 0. \quad (2.19)$$

Thus, from (2.8), (2.10), (2.12), (2.14), and (2.19), it is very easy to obtain that

$$\lim_{t \rightarrow \infty} f_1(t) = \infty, \quad (2.20)$$

$$\lim_{t \rightarrow \infty} f_2(t) = \infty, \quad (2.21)$$

$$\lim_{t \rightarrow \infty} f_3(t) = \infty, \quad (2.22)$$

$$\lim_{t \rightarrow \infty} f_4(t) = \infty. \quad (2.23)$$

From (2.14) and (2.19), it is clear that the function $f_4(t)$ is strictly increasing on $[1, \infty)$, and so, by virtue of (2.18) and (2.23), there exists a point $t_0 > 1$ such that $f_4(t) < 0$ on $[1, t_0]$ and $f_4(t) > 0$ on (t_0, ∞) . Hence, the function $f_3(t)$ is strictly decreasing on $[1, t_0]$ and strictly increasing on $[t_0, \infty)$. Similarly, by (2.17) and (2.22), there exists a point $t_1 > t_0 > 1$ such that $f_2(t)$ is strictly decreasing on $[1, t_1]$ and strictly increasing on $[t_1, \infty)$, and, by (2.11) and (2.21), there exists a point $t_2 > t_1 > 1$ such that $f_1(t)$ is strictly decreasing on $[1, t_2]$ and strictly increasing on $[t_2, \infty)$. Further, by (2.7), (2.9), and (2.20), there exists a point $t_3 > t_2 > 1$ such that $f(t)$ is strictly decreasing on $[1, t_3]$ and strictly increasing on $[t_3, \infty)$. Finally, by (2.3) and (2.16), it is deduced that the function $f(t)$ is negative on $(1, \infty)$. The inequality (2.1) is thus proved.

If $p = \mu = 1$, then the function (2.8) becomes

$$f_1(t) = (t - 1)^4 > 0 \quad (2.24)$$

for $t > 1$. Combining this with (2.7) and (2.5) results in that $f(t)$ is strictly increasing and positive on $(1, \infty)$. Therefore, the inequality (2.2) is obtained.

Combining the inequalities (2.1) and (2.2) with the monotonicity of $J(x)$ defined by (1.8), the double inequality (1.10) is established for all $\alpha \leq \lambda$ and $\beta \geq 1$.

For any given number p satisfying $1 > p > \lambda$, it is obvious that the limit (2.6) is positive. This positivity together with (2.3) and (2.4) implies that for $1 > p > \lambda$ there exists $T_0 = T_0(p) > 1$ such that the inequality

$$\overline{C}(pa + (1 - p)b, pb + (1 - p)a) > T(a, b)$$

holds for $\frac{a}{b} \in (T_0, \infty)$. This tells us that the constant λ is the best possible.

For $\frac{1}{2} < p < \mu = 1$, the quantity (2.13) is positive. Accordingly, there exists a number $\delta = \delta(p) > 0$ such that the function $f_3(t)$ is negative on $(1, 1 + \delta)$. This negativity together with (2.3), (2.5), (2.7) and (2.9) implies that for any $\frac{1}{2} < p < \mu = 1$, there exists $\delta = \delta(p) > 0$ such that the inequality

$$T(a, b) > \overline{C}(pa + (1 - p)b, pb + (1 - p)a)$$

is valid for $\frac{a}{b} \in (T_0, \infty)$. Consequently, the number μ is the best possible. The proof of Theorem 1.1 is complete.

3. PROOF OF THEOREM 1.2

In order to prove Theorem 1.2, we need the following Lemmas.

Lemma 3.1. *The Bernoulli numbers B_{2n} for $n \in \mathbb{N}$ have the property*

$$(-1)^{n-1} B_{2n} = |B_{2n}|, \quad (3.1)$$

where the Bernoulli numbers B_i for $i \geq 0$ are defined by

$$\frac{x}{e^x - 1} = \sum_{i=0}^{\infty} \frac{B_i}{i!} x^i = 1 - \frac{x}{2} + \sum_{i=1}^{\infty} B_{2i} \frac{x^{2i}}{(2i)!}, \quad |x| < 2\pi. \quad (3.2)$$

Proof. In [2, p. 16 and p. 56], it is listed that for $q \geq 1$

$$\zeta(2q) = (-1)^{q-1} \frac{(2\pi)^{2q}}{(2q)!} \frac{B_{2q}}{2}, \quad (3.3)$$

where ζ is the Riemann zeta function defined by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}. \quad (3.4)$$

From (3.3), the formula (3.1) follows. \square

Lemma 3.2. *For $0 < |x| < \pi$,*

$$\cot x = \frac{1}{x} - \sum_{n=1}^{\infty} \frac{2^{2n} |B_{2n}|}{(2n)!} x^{2n-1}. \quad (3.5)$$

Proof. This may be derived readily from combining the formula [1, p. 75, 4.3.70] with the identity (3.1). \square

Lemma 3.3. *For $0 < |x| < \pi$,*

$$\frac{1}{\sin^2 x} = \frac{1}{x^2} + \sum_{n=1}^{\infty} \frac{2^{2n} (2n-1) |B_{2n}|}{(2n)!} x^{2(n-1)}. \quad (3.6)$$

Proof. Since

$$\frac{1}{\sin^2 x} = \csc^2 x = -\frac{d}{dx}(\cot x),$$

the formula (3.6) follows from differentiating (3.5). \square

Now we are ready to prove Theorem 1.2. It is easy to see that the double inequality (1.12) is equivalent to

$$\alpha_1 < \frac{T(a, b) - A(a, b)}{C(a, b) - A(a, b)} < \beta_1. \quad (3.7)$$

Without loss of generality, we assume $a > b > 0$ and let $x = \frac{a}{b}$. Then $x > 1$ and

$$\frac{T(a, b) - A(a, b)}{C(a, b) - A(a, b)} = \frac{\frac{x-1}{2 \arctan \frac{x-1}{x+1}} - \frac{x+1}{2}}{\frac{x^2+1}{x+1} - \frac{x+1}{2}}.$$

Let $t = \frac{x-1}{x+1}$. Then $t \in (0, 1)$ and

$$\frac{T(a, b) - A(a, b)}{C(a, b) - A(a, b)} = \frac{\frac{t}{\arctan t} - 1}{t^2}.$$

Let $t = \tan \theta$ for $\theta \in (0, \frac{\pi}{4})$. Then

$$\frac{T(a, b) - A(a, b)}{C(a, b) - A(a, b)} = \frac{\frac{\tan \theta}{\theta} - 1}{(\tan \theta)^2} = \frac{\cot \theta}{\theta} - \frac{1}{\sin^2 \theta} + 1.$$

By Lemmas 3.2 and 3.3, we have

$$\begin{aligned} \frac{\cot \theta}{\theta} - \frac{1}{\sin^2 \theta} + 1 &= 1 - \sum_{n=1}^{\infty} \frac{2^{2n}}{(2n)!} |B_{2n}| \theta^{2n-2} - \sum_{n=1}^{\infty} \frac{(2n-1)2^{2n}}{(2n)!} |B_{2n}| \theta^{2n-2} \\ &= 1 - \sum_{n=1}^{\infty} \frac{n2^{2n+1}}{(2n)!} |B_{2n}| \theta^{2n-2} \end{aligned}$$

which is strictly decreasing on $(0, \frac{\pi}{4})$. Moreover, by L'Hôpital rule and standard argument, we have

$$\lim_{x \rightarrow 0^+} = \frac{1}{3} \quad \text{and} \quad \lim_{x \rightarrow (\pi/4)^-} = \frac{4}{\pi} - 1.$$

The proof of Theorem 1.2 is complete.

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